

JUL 16

To be presented at the Third Workshop on Structural Health Monitoring, Stanford, CA,  
September 12-14, 2001

## **ACTIVE WAVE PROPAGATION AND SENSING IN PLATES**

Anindya Ghoshal<sup>1</sup>, William N. Martin<sup>1</sup>, Mannur J. Sundaresan<sup>1</sup>, Mark J. Schulz<sup>2</sup>,  
and Frederick Ferguson<sup>1</sup>

**20011022 052**

---

<sup>1</sup>NASA Center For Aerospace Research, Department of Mechanical Engineering,  
North Carolina A&T State University, Greensboro, NC 27411, <sup>2</sup>Department of  
Mechanical Engineering, University of Cincinnati, Cincinnati, OH 45221

**DISTRIBUTION STATEMENT A**  
Approved for Public Release  
Distribution Unlimited

## **ABSTRACT**

Health monitoring of aerospace structures can be done using an active interrogation approach with diagnostic Lamb waves. Piezoelectric patches are often used to generate the waves, and it is helpful to understand how these waves propagate through a structure. To give a basic understanding of the actual physical process of wave propagation, a model is developed to simulate asymmetric wave propagation in a panel and to produce a movie of the wave motion. The waves can be generated using piezoceramic patches of any size or shape. The propagation, reflection, and interference of the waves are represented in the model. Measuring the wave propagation is the second important aspect of damage detection. Continuous sensors are useful for measuring waves because of the distributed nature of the sensor and the wave. Two sensor designs are modeled, and their effectiveness in measuring acoustic waves is studied. The simulation model developed is useful to understand wave propagation and to optimize the type of sensors that might be used for health monitoring of plate-like structures.

## **INTRODUCTION**

Active propagation of Lamb waves in the plane of the material is a common technique used to detect damage [1-7] in aerospace structures. Methods of measuring Acoustic Emissions (AE) are also used for damage detection [9-11]. The modeling of wave propagation is often done for plates that are of infinite dimension [13-18] because closed-form solutions of wave propagation in a bounded medium are difficult to obtain. A normal mode approach for obtaining a closed form solution for bending wave propagation in a plate is presented in [19, 20]. Finite-element methods are also used to model wave propagation, but the computation time is usually large and the representation of higher frequencies and modes are constrained by the number of elements used.

The normal mode approach is also used in the present paper to model

asymmetric Lamb wave or bending wave propagation in a simply supported plate. A composite material plate is studied here, and piezoceramic actuators and sensors are modeled on the plate for generating and receiving the waves. The objective of developing this model is to provide a tool to simulate wave propagation and to help the design of sensors to measure the waves. The model uses a closed-form solution and it is written in matrix and vector forms to run efficiently on a PC computer. With this technique, series and array configurations of sensors can be investigated including different shapes of sensors and actuators. Piezoelectric patches are used as the sensor and actuator material. The sensor design can mimic receptors that excite dendrites that are the inputs of neurons in the human nervous system. Ten or more nodes can be connected in a continuous single channel nerve. This continuous sensor approach is investigated first in the paper. Then, the same sensor elements used for the continuous sensor are connected in an N-by-N array that causes the individual signals from the sensors to be combined into 2N-1 array outputs, as compared to the single output from before. For large array sizes, this approach greatly reduces the number of channels of data acquisition instrumentation needed for structural health monitoring. A trade-off in these two approaches is that the continuous sensor is the simplest with only one channel of data acquisition, while the array uses more channels to more accurately locate damage. The modeling of the plate and the electrical modeling of the sensors are discussed next.

## MODELING THE PLATE

An algorithm to simulate waves propagating in a plate has been developed. The algorithm uses a step input to a simply supported plate, and it takes into account (integrates) the exact strains over the area of the sensor node to compute the voltage output. Details of the plate and the surface bonded actuator model will be given in a following journal paper. From the classical theory of vibration of plates, the equation of motion has been derived and written in terms of internal plate flexural moments  $M_x$ ,  $M_{xy}$ , and  $M_y$ , and actuator induced moments  $m_x$  and  $m_y$ . The equation of motion is then expressed in displacement form as:

$$D\nabla^4 w(x, y, t) + \rho h \ddot{w} = \frac{\partial^2 m_x}{\partial x^2} + \frac{\partial^2 m_y}{\partial y^2} \quad (1)$$

The actuator induced moments are expressed as Heaviside step functions. Using orthogonalization and separation of spatial and time variables, the temporal equation in terms of modes is derived. The total solution for the plate displacement is expressed in terms of a Fourier series (Navier solution) and by summation of modes for an impulse moment actuation:

$$w(x, y, t) = \sum_n \sum_m \frac{\bar{F}_{mn}}{\omega_{d_{mn}}} e^{-\zeta_{mn}\omega_{mn}t} \sin(\omega_{d_{mn}}t) \sin\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right) \quad (2)$$

where  $\omega_{d_{mn}} = \omega_{mn} \sqrt{1 - \zeta_{mn}^2}$ , and

$$\bar{F}_{mn} = \frac{4\Delta t}{\rho hab} (-1) [m_x^1 \frac{mb}{na} + m_y^1 \frac{na}{mb}] (\cos(\frac{m\pi x_1}{a}) - \cos(\frac{m\pi x_2}{a})) (\cos(\frac{n\pi y_1}{b}) - \cos(\frac{n\pi y_2}{b})) \quad (3)$$

where a and b are the width and breadth of the plate in x and y directions, m and n are the mode numbers, h is the thickness of the plate,  $m_x^1$  and  $m_y^1$  are the distributed surface moments,  $\rho$  is the mass density of the fiberglass plate, and z is h/2. The corresponding strains are:

$$\varepsilon_x(x, y, t) = -z \frac{\partial^2 w}{\partial x^2} = -z \sum_n \sum_m a_{mn}(t) \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b} \left( \frac{m\pi}{a} \right)^2 \quad (4)$$

$$\varepsilon_y(x, y, t) = -z \sum_n \sum_m a_{mn}(t) \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b} \left( \frac{n\pi}{b} \right)^2 \quad (5)$$

$$\gamma_{xy}(x, y, t) = 2z \frac{\partial^2 w}{\partial x \partial y} = 2z \sum_n \sum_m a_{mn}(t) \cos \frac{m\pi x}{a} \cos \frac{n\pi y}{b} \left( \frac{m\pi}{a} \right) \left( \frac{n\pi}{b} \right) \quad (6)$$

## MODELING OF THE SENSOR

The piezoceramic sensor nodes can be modeled as a capacitor in parallel with a current source. The piezoelectric constitutive equations used are listed in the IEEE standard ANSI/IEEE Std. 176-1987. These equations can be put into matrix form to give:

$$\begin{bmatrix} D \\ T \end{bmatrix} = \begin{bmatrix} \varepsilon^S & e \\ -e & c^E \end{bmatrix} \cdot \begin{bmatrix} E \\ S \end{bmatrix}, \begin{bmatrix} D \\ S \end{bmatrix} = \begin{bmatrix} \varepsilon^T & d \\ d & s^E \end{bmatrix} \cdot \begin{bmatrix} E \\ T \end{bmatrix}. \quad (7, 8)$$

The piezoceramic sensors can be modeled using the piezoelectric constitutive equations and by connecting the segments into an electric circuit as shown in Fig. 1. Please refer to [12] for the full derivation, but the expression for the output voltage equation for a sensor nerve is:

$$\frac{d}{dt}(i) + \frac{n \cdot i}{RC} = \frac{eA_e}{RC} \sum_{j=1}^n \dot{S}_j \quad (9)$$

where C is the capacitance of the PZT sensor, the effective capacitor area is  $A_e$ , the effective plate separation distance is h,  $i_c$  represents the component of the current going through the capacitor of the model, and  $i_g$  represents the component of the current generated by the piezoelectric sensor. The homogeneous and particular solutions for Equation (9) must be calculated and added for the total solution of the current  $i$ . The product of the current  $i(t)$ , and the impedance  $R$  of the measuring device equals the voltage of the series connected sensors as a function of time. Thus

we solve for the current to get the voltage  $V_o = iR$ . This voltage is proportional to the dynamic strain in the structure at the sensor and thus can be used to detect damage through dynamic strain measurements and acoustic emissions.

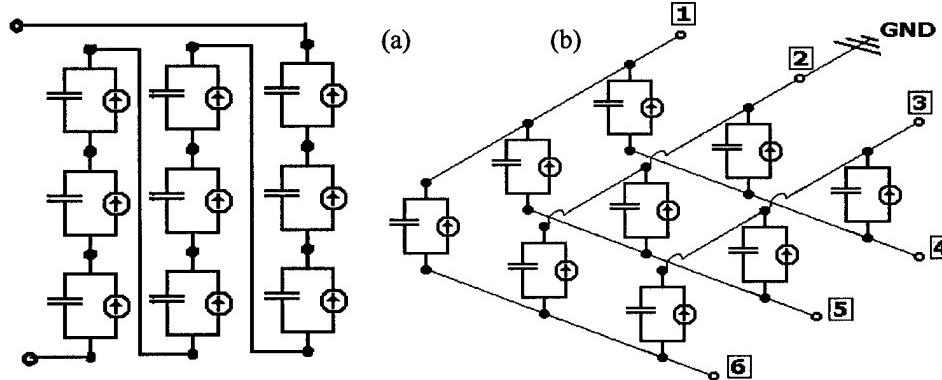


Figure 1. Circuit model of sensor (a) continuous sensor, (b) cross-sensor array.

## SIMULATION OF WAVE PROPAGATION

The simulation is performed using a model of the fiberglass plate. Three different cases of active wave propagation using a surface bonded PZT actuator and sensing using continuous and array sensors are presented here. The material properties of the fiberglass plate can be found in [11]. The size of the plate is 0.88m x 1.21m x 3.2mm. The first 100 vibration modes have been used for the simulation and the time step used is one micro-second.

Figure 2 shows the active wave propagation at the times 10 micro-sec to 360 micro-sec due to anti-symmetric Lamb waves generated by a PZT actuator placed at the center of the fiberglass plate. The actuator dimensions are 2.5cm x 5cm x 0.25mm. Figure 3 shows the voltage-time history of the continuous and the array sensors modeled on the plate.

Figure 4 shows the active wave propagation at the times 10 micro-sec to 360 micro-sec, due to anti-symmetric Lamb waves generated the PZT actuator placed at one edge of the fiberglass plate. The actuator dimensions are 2.5cm x 5cm x 0.25mm. Figure 5 shows the voltage-time history of the continuous and the array sensors modeled on the plate.

Figure 6 shows the active wave propagation at the times 10 micro-sec to 360 micro-sec, due to anti-symmetric Lamb waves generated the PZT actuator placed at the center and across the width of the fiberglass plate. The actuator dimensions are 0.81m x 2.5cm x 0.25mm. Figure 7 shows the voltage-time history of the continuous and the array sensors modeled on the plate.

Currently efforts are underway to model a crack in the plate and use the actuator and sensor bonded on the plate to detect the crack. Further investigations are being done to correlate the peaks in the voltage time history graphs with the time when the incident and the reflected waves hit the sensors, and when the waves are

scattered from a crack embedded in the plate. A neural network algorithm is currently being devised to detect and locate the crack in the plate model.

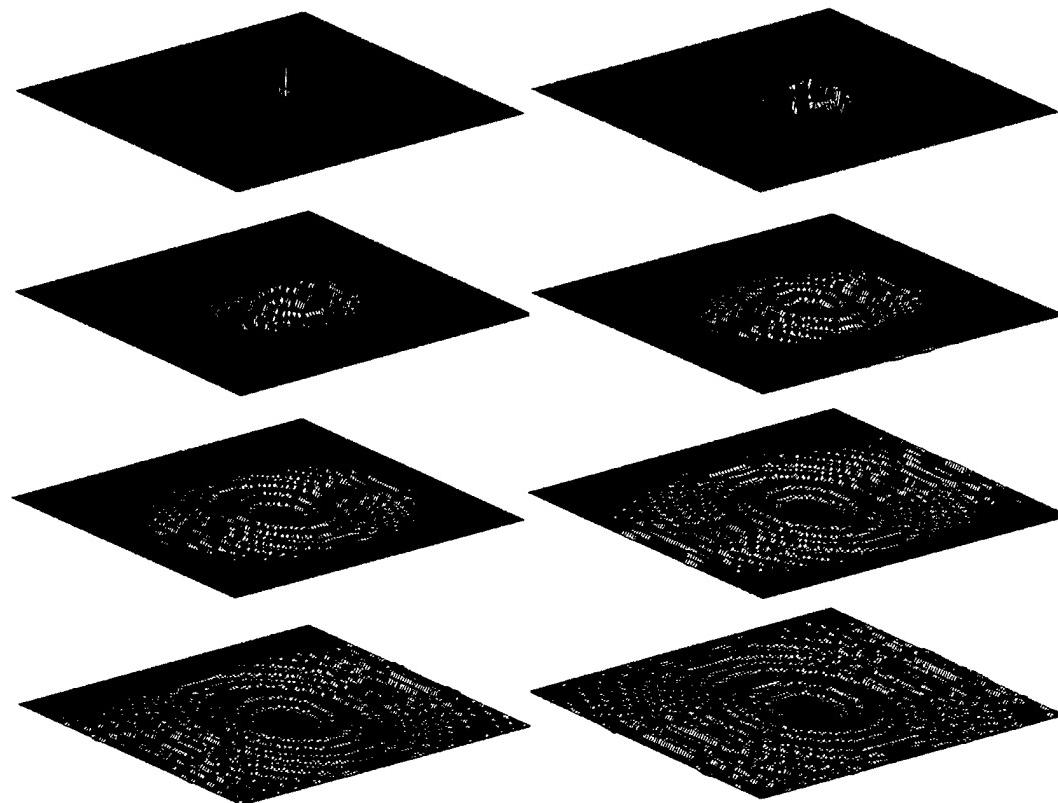


Figure 2. Wave propagation at 10, 60, 110, 160, 210, 260, 310, 360 micro-sec due to center actuation by a PZT patch.

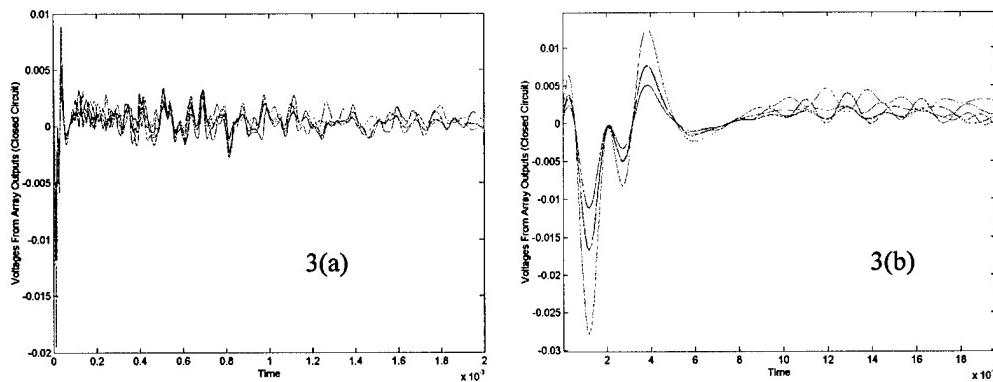


Figure 3. Voltage time history due to center actuation by a PZT patch; (a) array voltage for 2 ms, (b) array voltage for 0.2 ms.

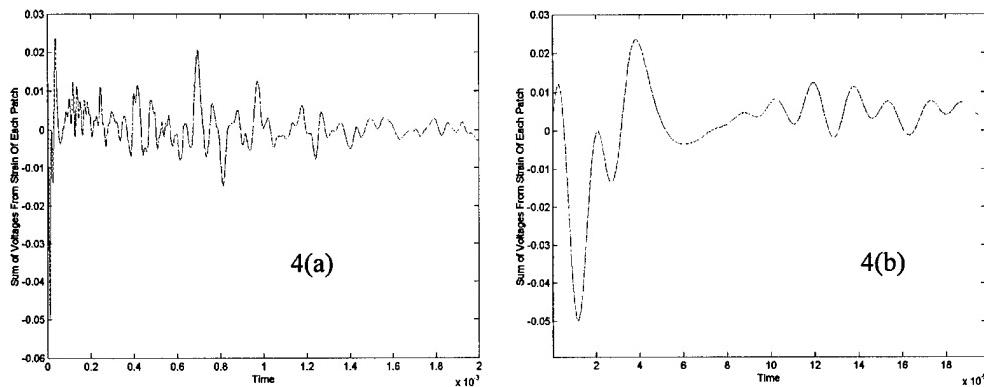


Figure 4. Voltage time history due to center actuation by a PZT patch; (a) continuous sensor voltage for 2 ms, (b) continuous sensor voltage for 0.2 ms.

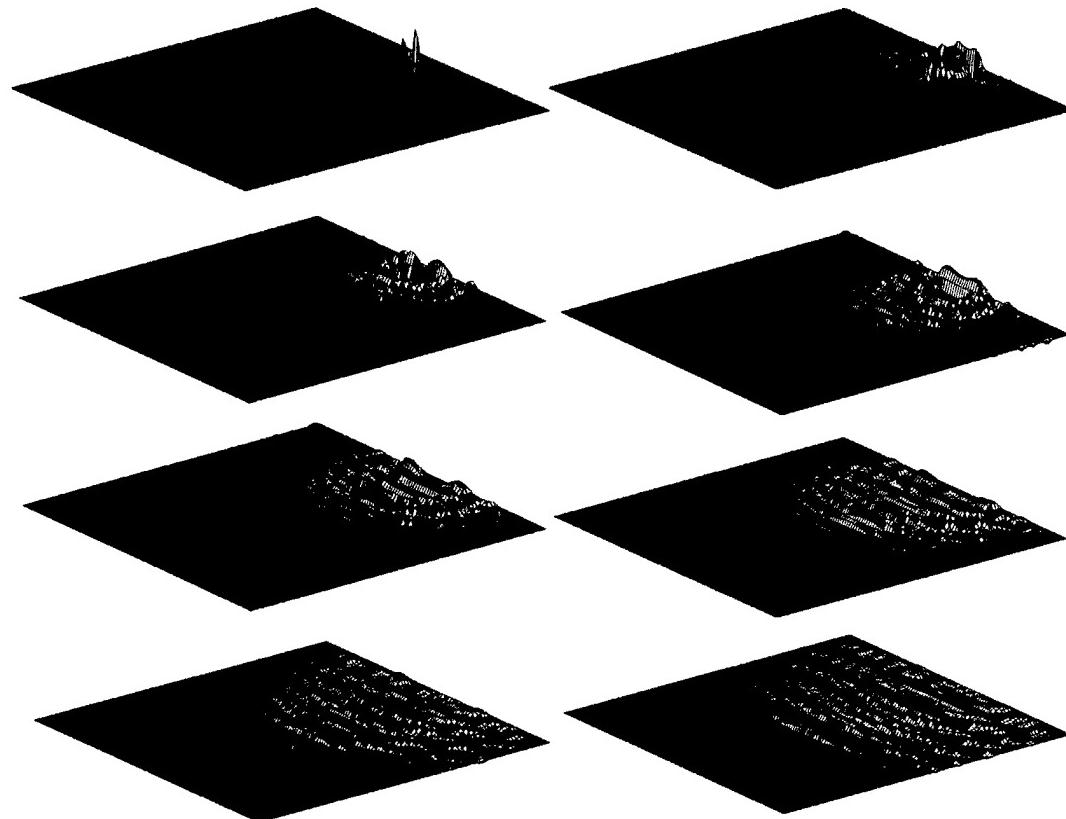


Fig 5. Wave propagation at 10, 60, 110, 160, 210, 260, 310, 360 micro-sec due to edge actuation by a PZT patch.

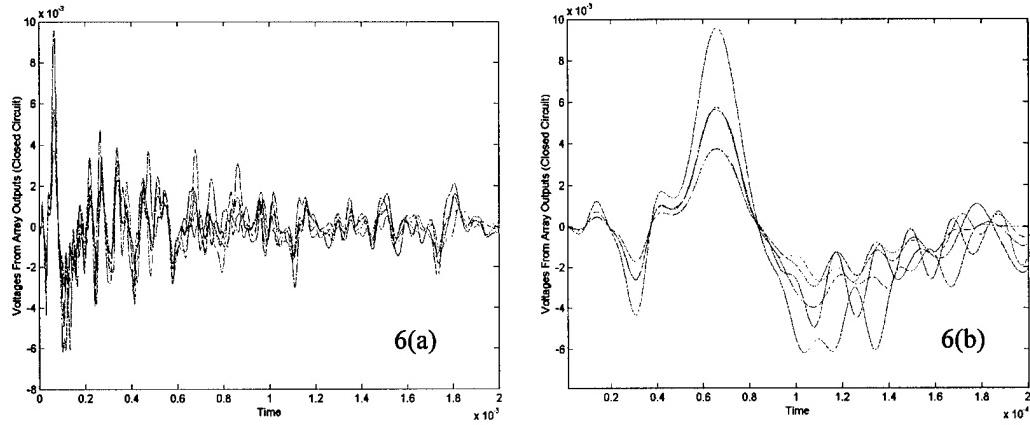


Figure 6. Voltage time history due to edge actuation by a PZT patch; (a) array voltage for 2 ms, (b) array voltage for 0.2 ms.

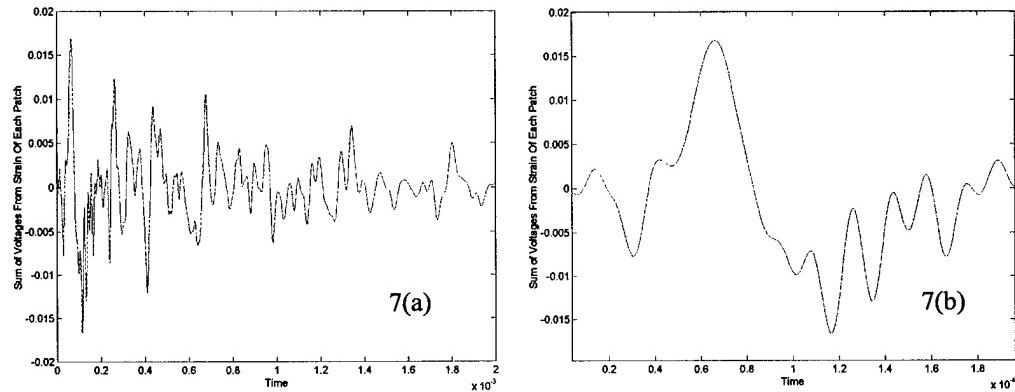


Figure 7. Voltage time history due to edge actuation by a PZT patch; (a) continuous sensor voltage for 2 ms, (b) continuous sensor voltage for 0.2 ms.

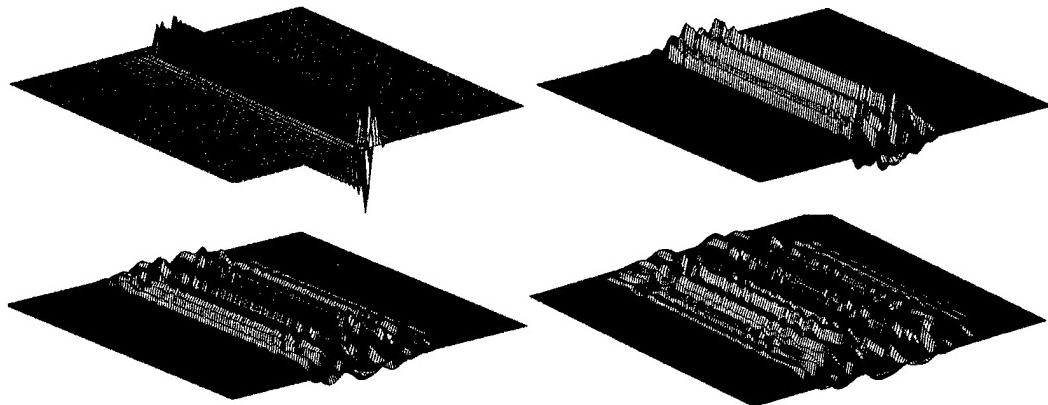


Figure 8, frames 1-4

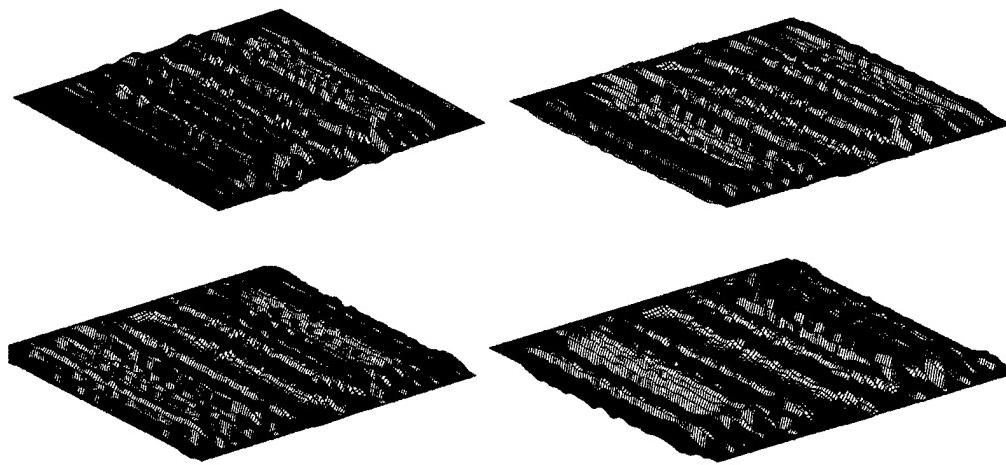


Figure 8 continued. Wave propagation at 10, 60, 110, 160, 210, 260, 310, 360 micro-sec due to center actuation by a wide PZT patch.

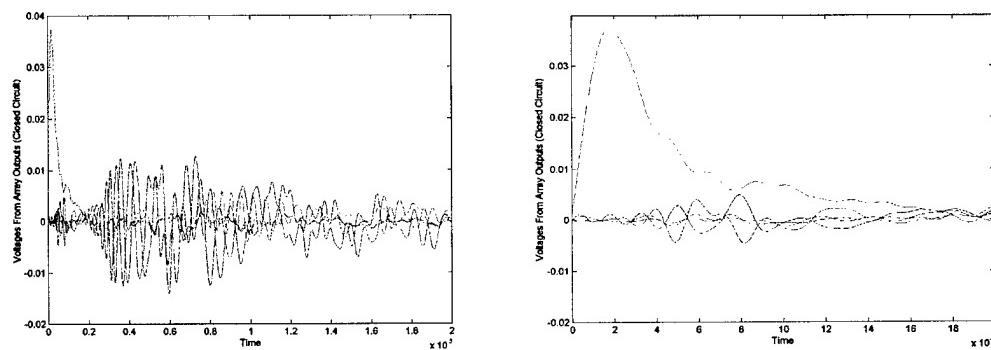


Figure 9. Voltage time history due to center actuation by a wide PZT patch; (a) array voltage for 2 ms, (b) array voltage for 0.2 ms.

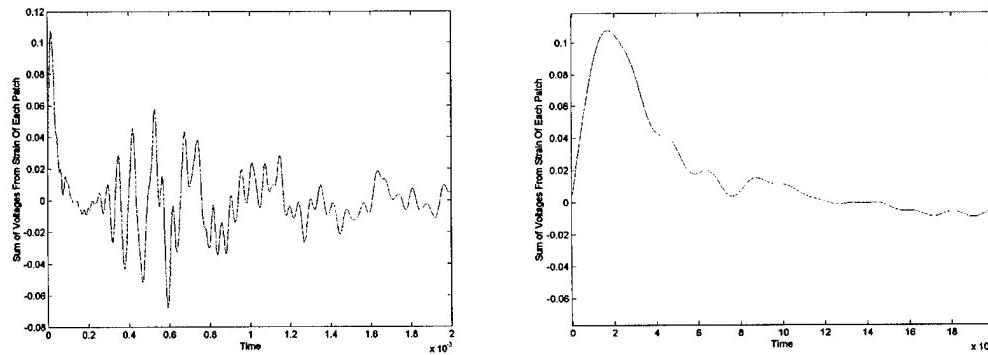


Figure 10. Voltage time history due to center actuation by a wide PZT patch; (a) continuous sensor voltage for 2 ms, (b) continuous sensor voltage for 0.2 ms.

## **CONCLUSIONS**

The simulations performed have shown that multiple piezoceramic patches can be connected together in a series or array pattern to simulate the way biological nerves have multiple inputs (dendrites) connected together. This reduces the number of channels of data acquisition needed to detect damage represented by acoustic emissions or high strains. The simulation model developed can be used to optimize the design of the neural system for different structural materials and sizes. This can be used for simulating active interrogation for damage detection in plate type structures. It helps in optimizing the actuator and sensor locations for both the continuous sensor nodes and for the sensor array configurations. The model can also be used for devising a neural network algorithm for detection, localization, and quantification of the crack/damage and monitoring the survivability of the plate structure.

## **ACKNOWLEDGMENT**

This material is based on research sponsored in part by the Air Force Research Laboratory under agreement number F49620-00-1-0232. The U.S. government is authorized to reproduce and distribute reprints for governmental purposes notwithstanding any copyright notation thereon. This material is also based on research sponsored in part by the U.S. Army Research Office under contract/grant number G DAAD 19-00-1-0536, and the NASA Marshall Space Flight Center under grant number NAG8-1646. The support for this research is gratefully acknowledged.

## **REFERENCES**

1. Pierce, S. G., Culshaw, B., Manson, G., Worden, K., Staszewski, W. J., 2000, "The application of ultrasonic Lamb wave techniques to the evaluation of advanced composite structures," Procs. of SPIE, Vol 3986, pp 93-103.
2. Allyne, D. N., and Cawley P., 1992, "Optimization of Lamb wave inspection techniques," J. Destructive Testing and Evaluation International, Vol. 25, Issue 1, pp 11-22.
3. Pierce, S. G., Philp, W. R., Culshaw, B., Gachagan A., McNab, A., Hayward G, and Lecuyer, F., 1996, "Surface-bonded optical sensors for the inspection of CFRP plates using ultrasonic Lamb waves, "Smart Materials and Structures, Vol 5, pp 776-787
4. Gachagan A., Hayward, G., McNab, A., Reynolds, P., Pierce, S. G., Philp, W. R., and Culshaw, B., 1999, "Generation and Reception of Ultrasonic guided waves in composite plates using conformable Piezoelectric Transmitters and Optical-Fiber detectors, " IEEE Trans. on Ultrasonics, Ferroelectrics, and Frequency Control, Vol 46, No. 1, pp. 72-81.
5. Rose, J. L., 2000, "Guided Wave Nuances for Ultrasonic Nondestructive Evaluation," IEEE Trans. on Ultrasonics, Ferroelectrics, and Frequency Control,

Vol 47, No. 3, May, pp. 575-582.

6. Wang, C., Chang, F., "Diagnosis of Impact Damage in Composite Structures with Built-In Piezoelectrics Network," *Proceedings of the SPIE*, Vol. 3990, p. 13, 2000.
7. Schwartz, W.G., Read, M.E., Kremer, M.J., Hinders, M.K., Smith, B.T., "Lamb Wave Tomographic Imaging System for Aircraft Structural Health Assessment," *SPIE Conference on NDE of Aging Aircraft, Airports, and Aerospace Hardware III*, Vol. 3586, P. 292, 1999, Newport Beach, CA.
8. M.J. Sundaresan, A., Ghoshal, M.J. Schulz, and C. Wilkerson, "Acoustic Emission Monitoring using Distributed Sensors," ASNT Spring Conference and 9th Annual Research Symposium, Birmingham, Alablama, March 27-31, 2000.
9. Sundaresan, M.J., Schulz, M.J., Ghoshal, A., Pratap, P., "A Neural System for Structural Health Monitoring," *SPIE 8<sup>th</sup> Int. Symposium on Smart Materials and Structures*, March 4-8, 2001.
10. Sundaresan, M.J., Ghoshal, A., and Schulz, M.J., "Sensor Array System," patent application, 6/00.
11. Ghoshal, A., Sundaresan, M.J., Schulz, M.J., Pai, P.F., "Continuous Sensors for Structural Health Monitoring," Adaptive Structures and Material Systems Symposium at the International Mechanical Engineering Congress and Exposition Winter Annual Meeting of the ASME, Nov. 5-10, 2000, Walt Disney World Dolphin, Orlando, Fla.
12. Schulz, M. J. Sundaresan, M.J., Ghoshal, A., Martin, W.N., "Evaluation of Distributed Sensors for Structural Health Monitoring," *ASME DTEC'01 Conf.*, Sept. 9-12, Pittsburgh, PA, 2001.
13. Doyle, J.F., 1997, *Wave Propagation in Structures*, 2<sup>nd</sup> Ed., Springer-Verlag, NY.
14. Achenbach, J.D., 1973, *Wave Propagation in Elastic Solids*, North Holland Publishing Company.
15. Ballentine, D.S., White, R.M., Martin, S.J., Ricco, A.J., Zellers, E.T., Frye, G.C., Wohltjen, H., 1997, *Acoustic Wave Sensors, Theory, Design and Physico-Chemical Applications*, Academic Press Inc.
16. Campbell, C.K., 1997, *Surface Acoustic Wave Devices for Mobile and Wireless Communications*, Academic Press.
17. Schmerr, L.W. Jr., 1998, *Fundamentals of Ultrasonic Nondestructive Evaluation*, Plenum Press.
18. Wasley, R.J., 1973, *Stress Wave Propagation in Solids*, Marcel Dekker, Inc, New York.
19. Gorman, M. R. and Prosser, W. H., "Application of Normal Mode Expansion to AE waves in Finite Plates," NASA Langley WebServer.
20. Prosser, W. H., "The propagation characteristics of the Plate Modes of Acoustic Emission Waves in thin Aluminum Plates and Thin Graphite/Epoxy Composites Plates and Tubes," Ph.D. dissertation, Johns Hopkins University, Baltimore, Maryland, 1991.